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DUCTILE TO BRITTLE TRANSITION IN SEA ICE UNDER UNIAXIAL LOADING

Abstract

A new constitutive model for sea ice, applicable to monotonic uniaxial loading in both compression and tension, is proposed. The stress-strain-strainrate behavior of sea ice is modelled accounting for strain softening and for fracture which manifests itself in terms of tensile cracking and crushing in compression. The model is used to predict first cracking in ice under uniaxial compressive loading based on a limiting tensile strain criterion and the results are calibrated with experimental data available in the literature.

1 INTRODUCTION

Field observations of sea ice indentation on offshore structures in the Arctic show that fracture processes are a major factor in ice-structure interaction.

The occurrence of first cracks in ice under compressive creep conditions in the laboratory has been studied by Gold /2/. Based on the assumption that grain boundary shear or sliding can be associated with a delayed elastic effect, Sinha /10/ postulated that

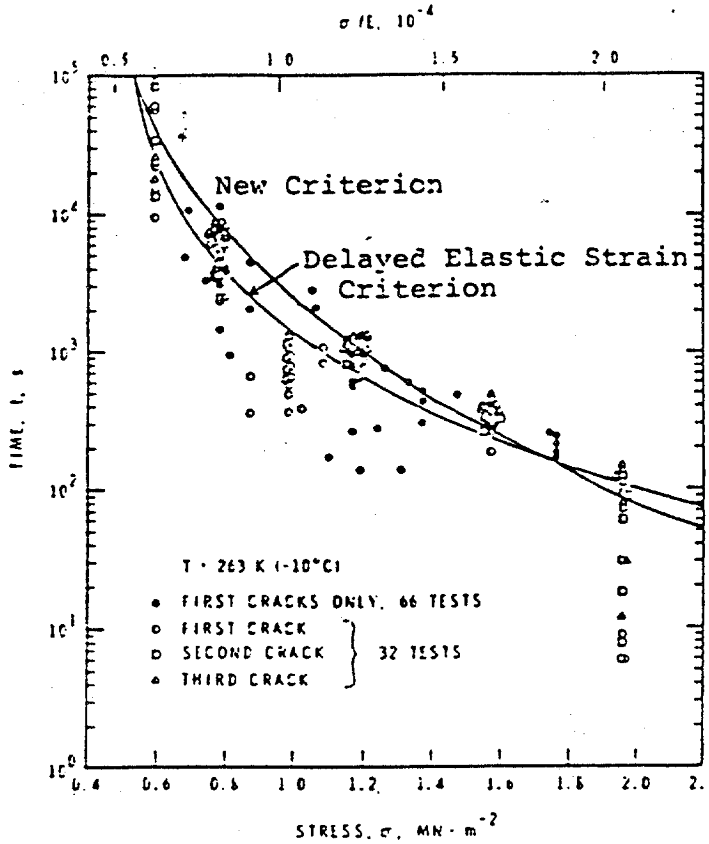

Fig. 1 Formation of first cracks during uniaxial compressive creep tests.

delayed elasticity can be linked to crack nucleation. With the help of his mathematical model for delayed elasticity and the experimental data of Gold, he showed that for S-2 ice of grain size 4.5 mm cracks begin to form if the delayed elastic strain exceeds 1.04×10^{-4}. The time to formation of first crack based on Gold's laboratory experiments and Sinha's delayed elastic strain criterion is plotted in Fig. 1.

A suddenly applied constant load case, i.e., creep, is not representative of loading conditions on offshore structures. A constant strainrate or stress-rate condition may be more realistic. Sanderson and Child /8/ consider typical stress-rates of 0.010-0.035 kPa s^{-1} and extreme stress-rates of 1-5 kPa s^{-1}. Using the principle of superposition, which Sinha /11/

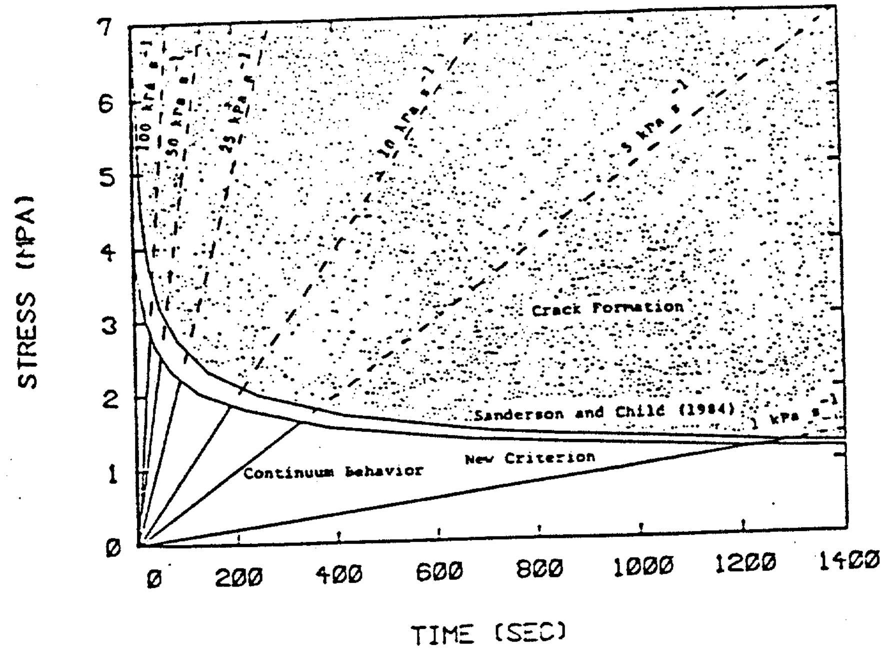

Fig. 2 Formation of first cracks during tests at constant stress-rate.

has shown to be valid for "icelike" materials under monotonically increasing stress, and the delayed elastic strain criterion they predict that first cracks in pure S-2 ice should typically occur at a stress of 0.6-0.7 MPa and in extreme conditions may occur at 1.3-1.8 MPa (Figs. 2 and 3). For sea ice, the stress levels are corrected by altering the net section stress due to brine volume as described by Sanderson /7/. The corresponding stresses are 0.4 MPa and 0.8-1.1 MPa (Fig. 4).

In order to explain the well-known discrepancy in ice forces between predictive models which use mechanical properties obtained in the laboratory and actual field measurements, Sanderson and Child /8/ propose that formation of first cracks in the field is synonymous with failure of the ice. As such, the stress levels identified in the previous paragraph are considered to limit ice forces. Although intuitively appealing, it is not clear how this failure criterion

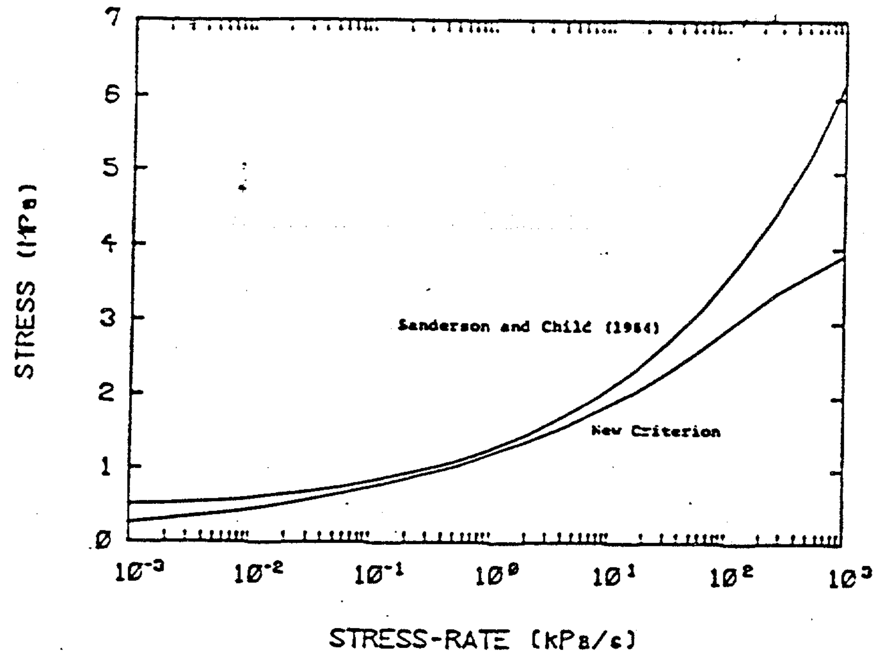

Fig. 3 Stress at which first cracks appear for pure ice at constant stress-rates.

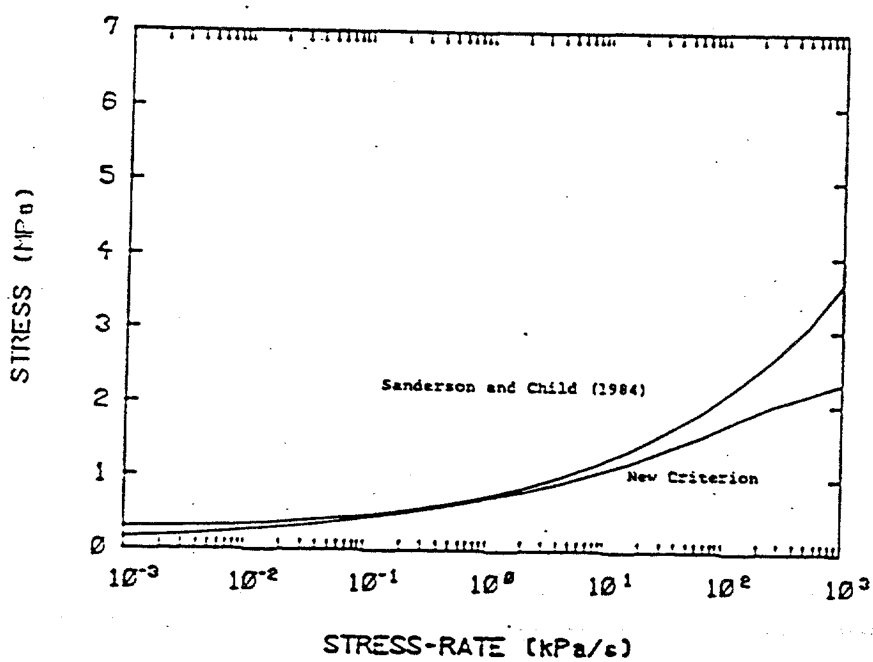

Fig. 4 Stress at which first cracks appear for sea ice at constant stress-rates.

can be incorporated in a finite element analysis framework for ice force prediction.

Fracture manifests itself in terms of tensile cracking and crushing in compression. Numerical analysis of ice-structure interaction processes in the creeping mode of deformation /1/ indicates that tensile stresses occupy a large fraction of the area of an ice sheet. Since ice is weaker in tension than in compression once cracks occur, accounting for the differing behavior of ice in tension may help to reduce ice force predictions significantly.

This paper presents a new constitutive model for sea ice, applicable to monotonic uniaxial loading in both compression and tension. The stress-strain-strainrate behavior of ice is modelled accounting for strain softening and fracture. The model is used to predict the occurrence of first cracks in ice under uniaxial compressive loading. Tensile strains occur under this loading condition as a result of the Poisson effect or incompressibility condition. Once cracks occur, the material continues to sustain compressive load but loses its ability to carry tensile loads in the transverse direction if applied. This is a realistic assumption and is often used in modeling concrete behavior /12/. A limiting tensile strain criterion dependent on the instantaneous strainrate in tension is developed to predict crack nucleation. The results for compressive creep compare very well with the experimental data of Gold /2/.

2 NEW UNIAXIAL CONSTITUTIVE MODEL

A phenomelogical approach based on simple thermo-rheological models has been used in developing the new uniaxial constitutive model /13/. The model is based on Orowan's concept /5/ that material strength is affected simultaneously by work hardening or strain hardening and work softening or recovery. Fracture in pure ice is modelled using the 'strength'-strainrate data of Ashby and Cooksley contained in Palmer et al. /6/. The resulting model is consistent with Michel's /4/ schematic idealization of ice behavior.

For constant strainrates of up to $5x10^{-4}$ s^{-1} under compressive loading, the stress-strain-strainrate behavior of ice is given by:

$$\sigma = \frac{A}{M} \dot{\varepsilon}^{\frac{1}{N}} [1-\exp(-M\varepsilon)] - \frac{B}{L} \dot{\varepsilon}^{\frac{1}{K}-1} [1-\exp(-L\varepsilon)] \qquad (1)$$

where A=114025 MPa $s^{1/3}$, B=217408 MPa $s^{2/3}$, M=1411.2, L=430, N=3 and K=0.6 based on Wang's /14/ experimental data for sea ice. At strainrates greater than 10^{-2} s^{-1}, pure ice is assumed to fracture (crush) at a stress of 5.1 MPa and to behave as a linear elastic material with Young's modulus, E, equal to 9.5 GPa. The intermediate strainrates define the ductile-to-brittle transition in compression. For strainrates between $5x10^{-4}$ and 10^{-3} s^{-1}, the fracture strength is assumed to be 7.14 MPa and the stress-strain behavior up to fracture is given by Eq. (1). For strainrates in the range 10^{-3}-10^{-2} s^{-1}, a linear interpolation between 7.14 and 5.1 MPa on log-log scale is used to define the fracture strength, while the stress-strain behavior up to fracture is defined by Eq. (1) for a strainrate of 10^{-3} s^{-1} (the initial tangent modulus of Eq. (1) reaches a value of 9.5 GPa, the linear elastic modulus, at this strainrate).

Constant stress-rate and creep curves in compression generated with this model agree with Wang's

/14/ theoretical model and the experimental data used by him for calibration purposes. Moreover, the adopted modeling strategy avoids the numerical problems encountered by Wang.

The stress-strain-strainrate behavior in uniaxial tension prior to fracture is considered to be identical to that under uniaxial compression as given by Eq. (1). Hawkes and Mellor /3/ justify this assumption for creep data. For strainrates less than 3×10^{-8} s^{-1} (i.e., tension 'strength' of 0.42 MPa for pure ice and 0.25 MPa for sea ice), ice does not fracture in tension. For strainrates greater than 5×10^{-5} s^{-1}, tensile cracking occurs at a stress of 2.04 MPa. A linear interpolation on log-log scale is used for intermediate strainrates, defining the ductile-to-brittle transition in tension.

3 PREDICTION OF FIRST CRACK OCCURRENCE

The prediction of first crack occurrence under uniaxial compressive creep and constant stress-rate conditions is considered here. In order to make this prediction, it is necessary to monitor the tensile strains resulting from the Poisson effect or incompressibility condition and to compare them with the strain for tensile fracture at the instantaneous strainrate. When the actual instantaneous tensile strain becomes equal to the instantaneous fracture strain the first crack is assumed to occur. This is the limiting tensile strain criterion for crack nucleation. A numerical procedure is developed to make the crack prediction. A time increment not exceeding 10^{-5} divided by the instantaneous strainrate is necessary to obtain accurate results.

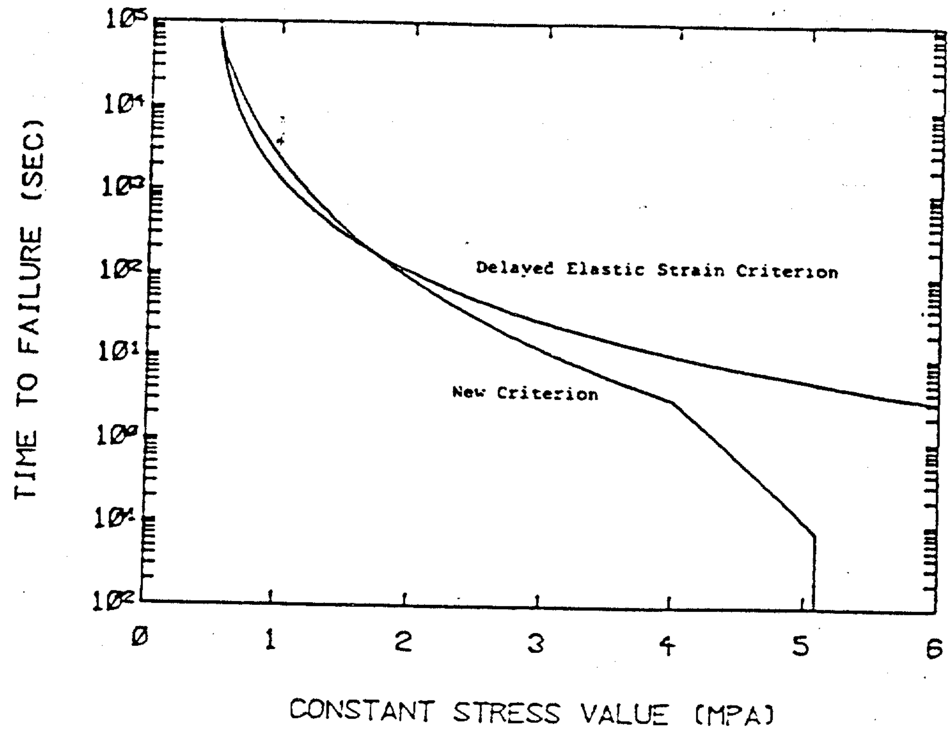

Fig. 5 Formation of first cracks in uniaxial creep up to the fracture stress in compression.

Figure 1 contains the prediction of first cracks using the limiting tensile strain criterion under creep conditions. Comparison with the experimental data of Gold shows that the proposed criterion is in excellent agreement with data. In particular, the time to first crack asymptotically approaches infinity as the compressive stress reduces to 0.52 MPa. The choice of stress-strainrate at which ice transits from ductile to fracture behavior in tension , i.e., 0.42 MPa and 3×10^{-8} s^{-1},defines this asymptote. The large scatter in the experimental results at σ=2 MPa is possibly due to the finite rise time of two seconds for the applied load to reach the constant stress state for ideal creep. The limiting tensile strain criterion compares well with the delayed elastic strain criterion of Sinha for the range of stresses considered in the figure. However, at higher stresses the two criteria are in significant disagreement (Fig. 5). At the compressive fracture stress of 5.1 MPa, the proposed model predicts a zero time to first crack since no creep can occur under this loading. At stresses greater than about 4 MPa which corresponds to the leveling-off of the tensile fracture stress to 2.04 MPa, the tensile strain to fracture reduces faster than at lower stresses. This leads to a faster reduction in time to first crack. The sharp kink at the 4 MPa transition point can be eliminated by a smoother transition in the cracking criterion around the strainrate of $5 \times 10^{-5}\ s^{-1}$.

Figure 2 contains the prediction of first cracks under constant stress-rate conditions. The agreement between Sanderson and Child's analysis based on the delayed elastic strain criterion and the present criterion is very good in general. At infinite stress-rate, the stress at first crack is limited by the compressive fracture stress of 5.1 MPa. This is predicted by the proposed fracture criterion. Figures 3 and 4 show that for typical stress-rates the stress at first crack is 0.45-0.60 MPa for pure ice and 0.26-0.35 MPa for sea ice. For extreme stress-rates the corresponding numbers are 1.2-1.6 MPa and 0.7-0.9 MPa.

4 CONCLUSIONS

This paper has proposed a new uniaxial constitutive model for sea ice that accounts for strain softening and fracture (cracking and crushing). The adequacy of the model has been demonstrated by comparison with experimental data obtained under constant strainrate, creep, and constant stress-rate conditions. A limiting tensile strain criterion has been postulated to predict first cracks in ice and its validity has been established by comparison with available experimental data.

The constitutive model is being extended to account for unloading and reloading conditions and for multiaxial stress states. The resulting model will be incorporated in a finite element analysis framework to predict indentation pressures and forces using the limiting tensile strain criterion for crack initiation and propagation. For load transmitting systems such as ice features (as opposed to load bearing structural systems) a limiting tensile strain criterion for fracture propagation is likely to be conservative when compared to a classical fracture mechanics approach. This is because the latter considers only the propagation of pre-existing cracks with a given distribution of sizes, while the former may be used to predict both the initiation and propagation of cracks in a material originally in virgin (flawless) form.

5 ACKNOWLEDGEMENTS

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